1. Consider the category Top_* of pointed topological spaces and the category Grp of groups. Then, for $1 \le n \in \mathbb{N}$, the fundamental groups π_n act functorially on pointed spaces and continuous functions. In other words, the fundamental groups are functors
$$\pi_n : \mathsf{Top}_* \to \mathsf{Grp}$$
2. Consider the categories: (a) Top of topological spaces, (b) Ch of chain complexes and chain maps and (c) Ab of abelian groups and homomorphisms. Then, for any $n \in \mathbb{N}$, the homology functors $H_n : \mathsf{Top} \to \mathsf{Ab}$ can actually be considered to be the composite $H_n = H_n \circ C_*$ of two functors:
   (a) The functor $C_* : \mathsf{Top} \to \mathsf{Ch}$ taking a space X to the chain complexes generated by simplices $\Delta^n \to X$ and continuous functions to chain maps defined by composition,
   (b) The algebraic functor $H_n : \mathsf{Ch} \to \mathsf{Ab}$ taking a chain complex to the associated homology group and chain maps to the associated group hom.
3. For the category Man_*^p of pointed class C^p manifolds and class C^p maps preserving the points, there is a functor $d : \mathsf{Man}_*^p \to \mathsf{Vect}_{\mathbb{R}}$ taking a manifold to the tangent space at its dedicated point and a differentiable map to its derivative:
$$d(M, x_0) := TM_{x_0}$$
$$d(f : (M, x_0) \to (N, y_0)) := df_{x_0} : TM_{x_0} \to TN_{y_0}$$
   Note that one needs to do more work to see the functoriality, but the whole reason this is a functor is because the Chain Rule provides well-definedness for composition!
4. Let $f : A \to B$ be a function between sets. The (covariant) power-set functor $\mathscr{P} : \mathsf{Set} \to \mathsf{Set}$ takes sets A to their powersets $\mathscr{P}A$, and takes functions $f : A \to B$ to direct image functions
$$\mathscr{P}f : \mathscr{P}A \to \mathscr{P}B, \ U \subset A$$
$$\mathscr{P}f(U) := \{f(u) \mid u \in U\} \subset B$$
5. For any (locally-small) category $\mathbf{C}$ and any object $c \in \mathbf{C}$, the hom-functors $\mathrm{Hom}(c, -) : \mathbf{C} \to \mathsf{Set}$ takes objects and morphisms to
$$\mathrm{Hom}(c, a) := \{f : c \to a \mid f \in \mathbf{C}\}$$
$$\mathrm{Hom}(c, g : a \to b) := g_*(f : c \to a) = g \circ f : c \to a \to b$$